Core Content

Cluster Title: Solve real-world and mathematical problems involving area, surface area, and volume.

**Standard 2:** Find the volume of a right rectangular prism with appropriate unit fraction edge lengths by packing it with cubes of the appropriate unit fraction edge lengths (e.g., $3\frac{1}{2} \times 2 \times 6$) and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = Bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Clarification: It is not intended that this be modeled physically; it should be a conceptual activity modeled with drawings and diagrams. (Note: This standard is worded differently than the official standard. There are typos and other errors in the official version. They are corrected here and a clarification is included.)

**MASTERY Patterns of Reasoning**

**Conceptual:**
- Measuring with fractional units requires students to relate volume to multiplication with fractions.
- Describe the impact of defining volume by fractional factors.

**Procedural:**
- Use these formulas interchangeably, $V = lwh$ and $V = Bh$.
- Make the connection that when finding volume $l \times w$ is the same as $B$.

**Representational:**
- Composing whole cubes with fractional unit cubes
- Prove that the volume formula works by creating diagrams of prisms with unit fraction edge lengths and showing how unit fraction cubes pack them.

**Supports for Teachers**

**Critical Background Knowledge**

**Conceptual:**
- Volume is measured with cubic units.
- The nature of volume as an attribute.
- Since volume is a different attribute it requires a different measurement unit: cubic units.
- Prisms are three-dimensional.
- Volume is filling a prism.
**Procedural:**  
- Ability to multiply fractions  
- Finding the area of polygons, including those with unit fraction edge lengths  
- Substitution for values in formulas  
- Finding volume of prisms with whole unit side lengths

**Representational:**  
- Use of physical models with whole-unit side lengths.  
- Find volume using a unit cube model.

**Academic Vocabulary**  
- volume, rectangular prism, length, width, height, base, cubic units, fraction edge length, unit fraction

**Instructional Strategies Used**

- Review of 5.MD.5:  
  - Explore with cubes and arrange them into layers to create rectangular prisms. Record the dimensions of the first/base layer, add a second layer, determine new dimensions, and look for patterns to predict what will happen when a third layer is added. Add the third layer and determine if your prediction was correct. Make connections to formulas.  
  - Hold up a cube and explain that the edge measures one unit and that is the standard for finding the volume of a solid figure. The volume of a solid figure is the number of same sized cubes filling the space so that there are no gaps and overlaps.  
  - Make nets of rectangular prisms on graph paper. Fold and determine volume.  
- Define one cubic unit in order to see fractional parts. See resource.  
- Apply to formula using fractional edge lengths.

**Resources Used**
**Mathematical Task:**
Jaime has the following rectangular prisms (boxes) that he would like to send to his friend Carla through the mail:

- Box 1 – 1 inches long by 4 inches wide by 6 inches height
- Box 2 – 1½ inches long by 4 inches wide by 6 inches height
- Box 3 – 3 inches long by 4 inches long by 6 inches height
- Box 4 – 1½ inches long by ½-inch wide by 6 inches height

Process between each step of the problem. How did the answer to one box lead to the answer for the next? Describe the effect of fractional edge lengths. What is the total volume of the boxes? Determine the dimensions of the smallest possible box that Jaime could use to send the 4 boxes to Carla in one shipment. How much empty space will there be? Prove your answer by drawing a representation on grid paper or constructing the boxes.

**Assessment Tasks Used**

<table>
<thead>
<tr>
<th>Skill-based Task:</th>
<th>Problem Task:</th>
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<tbody>
<tr>
<td>A flower box is 3 ft. long 2¾ ft. wide and ½ ft. deep. How many cubic feet of dirt can it hold?</td>
<td>Build 3 rectangular prisms with the volume of 36 cubic units. At least one of the side lengths of each prism is a fractional unit. What are the dimensions of each of the rectangular prisms you built?</td>
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<td>A gallon of water uses 231 cubic in. of space. How many gallons of water are needed to fill this aquarium?</td>
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<table>
<thead>
<tr>
<th>Draw a diagram to match:</th>
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</thead>
<tbody>
<tr>
<td>l = 12½ in.</td>
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<tr>
<td>w = 8¼ in.</td>
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<tr>
<td>h = 12½ in.</td>
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