Core Content

Cluster Title: Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard 3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.

MASTERY Patterns of Reasoning

Conceptual:

Know that the distributive property has limitations in division ($\frac{3+5}{2} = \frac{3}{2} + \frac{5}{2}$ but $\frac{14}{2+5} \neq \frac{14}{2} + \frac{14}{5}$)

Understand that the properties used with numbers also apply to expressions with variables.

Procedural:

Apply the properties of operations with expressions involving variables to generate equivalent expressions.

Representational:

Use manipulatives or diagrams to represent the distributive property.
Show properties of operations to be equivalent with manipulatives, diagrams or story contexts.

Supports for Teachers

Critical Background Knowledge

Conceptual:

Variables are letters that stand for numbers.
Understanding properties of operations and applying each of them in numeric representations (e.g., $3 + 2 = 2 + 3$). Multiplication is repeated addition.
Understand the associative property of addition and multiplication, the commutative property of addition and multiplication, the identity property of addition and multiplication and the distributive property of multiplication over addition or subtraction.
Recognize how to use common calculations (such as fact families, basic math facts, number bonds, composing and decomposing numbers) to generate solutions to problems.
Procedural:
  Find common factors of two whole numbers.
Representational:
  Model operations of addition, subtraction, multiplication and division.

Academic Vocabulary and Notation
\( x \), \( x \cdot \frac{1}{x} \), \( x(a + b) \), associative property, commutative property, distributive property, equivalent, identity property, identity element, variable

Instructional Strategies Used
Students must be able to make connections between what they already know about the properties of operations when used with numbers and how those properties apply when used with algebraic notation (using variables). Therefore, teachers should first present the topic of equivalent expressions using numbers and then shift to variables.

Have students open a package of M&Ms and use the distributive property to write expressions showing how many of each color would be in 5 bags, 10 bags etc. (e.g. There are 10 red, 15 green, and 12 yellow in one bag. That could be represented by the expression \( 10r + 15g + 12y \). To represent the number in 5 bags you would obtain the expression \( 5(10r + 15g + 12y) \). Using the distributive property, the expression is \( 50r + 75g + 60y \). So there are 50 red, 75 green, and 60 yellow M&Ms in 5 bags.)

Assessment Tasks Used

Skill-based Task:
Generate an equivalent expression for each of the following:
- \( 4(x - 2) \)
- \( 15x - 24y \)
- \( x + x + y + y \)
- \( 5x + 2y \)
- \( 5r + (2s + 2t) \)

Problem Task:
In one packet of nuts, there are two different types of nuts. There are 5 peanuts \( (p) \) and 7 cashews \( (c) \) in each container. I have 6 packets of nuts; write two expressions that show how many nuts I have all together.

Possible answers: \( 6(5 + 7) \) or \( (6 \times 5) + (6 \times 7) \)

Code: 6.EE
Sarah says that the two expressions $3(2 + x)$ and $6 + x$ are equivalent. Is she right? If not, explain.

Correct response: No, that is not correct. $6 + 3x$ is the correct response since the 3 must be distributed through both terms in the parentheses.